

1-7

Midpoint and Distance in the Coordinate Plane

Common Core State Standards

Prepares for G-GPE.B.4 Use coordinates to prove simple geometric theorems . . .

Prepares for G-GPE.B.7 Use coordinates to compute perimeters . . . and areas . . . **Also G-GPE.B.6**
MP 1, MP 3, MP 4

Objectives To find the midpoint of a segment
To find the distance between two points in the coordinate plane



Try drawing the situation on graph paper if you are having trouble visualizing it.

SOLVE IT!

Getting Ready!

In a video game, two ancient structures shoot light beams toward each other to form a time portal. The portal forms exactly halfway between the two structures. Your character is on the grid shown as a blue dot. How do you direct your character to the portal? Explain how you found your answer.

LEFT

UP

RIGHT

DOWN



MATHEMATICAL PRACTICES

In this lesson, you will learn how to find midpoints and distance on a grid like the one in the Solve It.

Essential Understanding You can use formulas to find the midpoint and length of any segment in the coordinate plane.



Key Concept Midpoint Formulas

Description

On a Number Line

The coordinate of the midpoint is the *average* or *mean* of the coordinates of the endpoints.

In the Coordinate Plane

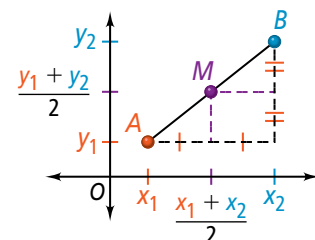
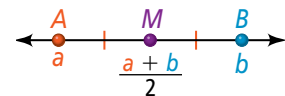
The coordinates of the midpoint are the average of the x -coordinates and the average of the y -coordinates of the endpoints.

Formula

The coordinate of the midpoint M of \overline{AB} is $\frac{a + b}{2}$.

Given \overline{AB} where $A(x_1, y_1)$ and $B(x_2, y_2)$, the coordinates of the midpoint of \overline{AB} are $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Diagram



Think

Which Midpoint Formula do you use?
If the endpoints are real numbers, use the formula for the number line. If they are ordered pairs, use the formula for the coordinate plane.



Problem 1 Finding the Midpoint

- A** \overline{AB} has endpoints at -4 and 9 . What is the coordinate of its midpoint?

Let $a = -4$ and $b = 9$.

$$M = \frac{a + b}{2} = \frac{-4 + 9}{2} = \frac{5}{2} = 2.5$$

The coordinate of the midpoint of \overline{AB} is 2.5 .

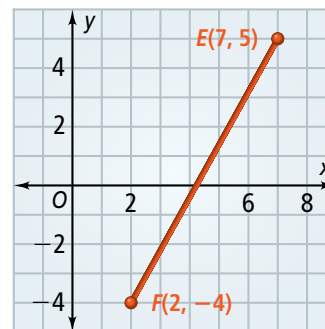
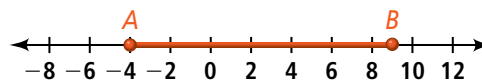
- B** \overline{EF} has endpoints $E(7, 5)$ and $F(2, -4)$. What are the coordinates of its midpoint M ?

Let $E(7, 5)$ be (x_1, y_1) and $F(2, -4)$ be (x_2, y_2) .

$$x\text{-coordinate of } M = \frac{x_1 + x_2}{2} = \frac{7 + 2}{2} = \frac{9}{2} = 4.5$$

$$y\text{-coordinate of } M = \frac{y_1 + y_2}{2} = \frac{5 + (-4)}{2} = \frac{1}{2} = 0.5$$

The coordinates of the midpoint of \overline{EF} are $M(4.5, 0.5)$.



- Got It?** 1. a. \overline{JK} has endpoints at -12 and 4 on a number line. What is the coordinate of its midpoint?
b. What is the midpoint of \overline{RS} with endpoints $R(5, -10)$ and $S(3, 6)$?

When you know the midpoint and an endpoint of a segment, you can use the Midpoint Formula to find the other endpoint.



Problem 2 Finding an Endpoint

- The midpoint of \overline{CD} is $M(-2, 1)$. One endpoint is $C(-5, 7)$. What are the coordinates of the other endpoint D ?

Let $M(-2, 1)$ be (x, y) and $C(-5, 7)$ be (x_1, y_1) . Let the coordinates of D be (x_2, y_2) .

$$(-2, 1) = \left(\frac{-5 + x_2}{2}, \frac{7 + y_2}{2} \right)$$

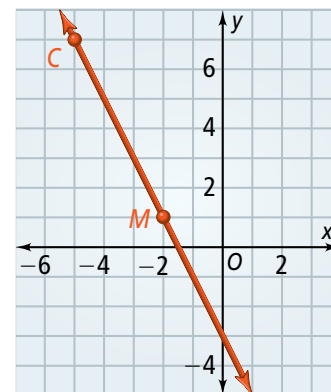
\swarrow x y \searrow

$$-2 = \frac{-5 + x_2}{2} \quad \text{Use the Midpoint Formula.} \quad 1 = \frac{7 + y_2}{2}$$

$$-4 = -5 + x_2 \quad \text{Multiply each side by 2.} \quad 2 = 7 + y_2$$

$$1 = x_2 \quad \text{Simplify.} \quad -5 = y_2$$

The coordinates of D are $(1, -5)$.



- Got It?** 2. The midpoint of \overline{AB} has coordinates $(4, -9)$. Endpoint A has coordinates $(-3, -5)$. What are the coordinates of B ?

Plan

How can you find the coordinates of D ?
Use the Midpoint Formula to set up an equation. Split that equation into two equations: one for the x -coordinate and one for the y -coordinate.

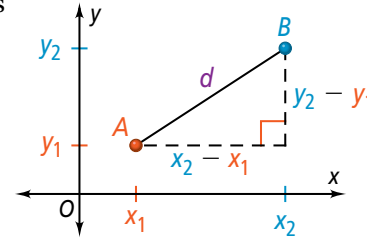
In Lesson 1-3, you learned how to find the distance between two points on a number line. To find the distance between two points in a coordinate plane, you can use the Distance Formula.



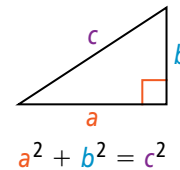
Key Concept Distance Formula

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



The Distance Formula is based on the *Pythagorean Theorem*, which you will study later in this book. When you use the Distance Formula, you are really finding the length of a side of a right triangle. You will verify the Distance Formula in Chapter 8.



Problem 3 Finding Distance

GRIDDED RESPONSE

What is the distance between $U(-7, 5)$ and $V(4, -3)$? Round to the nearest tenth.

Let $U(-7, 5)$ be (x_1, y_1) and $V(4, -3)$ be (x_2, y_2) .

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Use the Distance Formula.}$$

$$= \sqrt{(4 - (-7))^2 + (-3 - 5)^2} \quad \text{Substitute.}$$

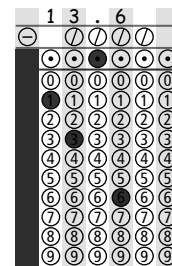
$$= \sqrt{(11)^2 + (-8)^2} \quad \text{Simplify within the parentheses.}$$

$$= \sqrt{121 + 64} \quad \text{Simplify.}$$

$$= \sqrt{185}$$

$$185 \rightarrow 13.60147051 \quad \text{Use a calculator.}$$

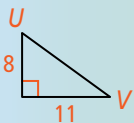
To the nearest tenth, $UV = 13.6$.



Think

What part of a right triangle is \overline{UV} ?

\overline{UV} is the hypotenuse of a right triangle with legs of length 11 and 8.



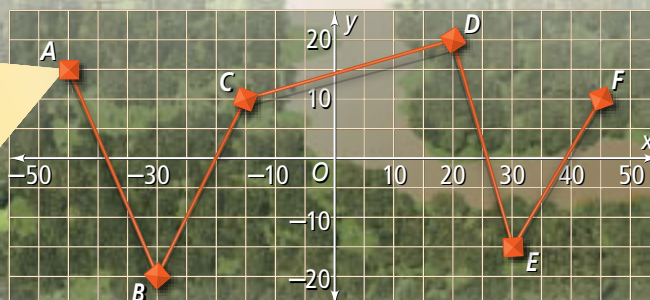
Got It? 3. a. \overline{SR} has endpoints $S(-2, 14)$ and $R(3, -1)$. What is SR to the nearest tenth?

b. **Reasoning** In Problem 3, suppose you let $V(4, -3)$ be (x_1, y_1) and $U(-7, 5)$ be (x_2, y_2) . Do you get the same result? Why?



Problem 4 Finding Distance

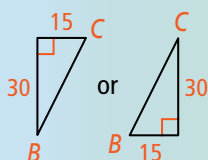
Recreation On a zip-line course, you are harnessed to a cable that travels through the treetops. You start at Platform A and zip to each of the other platforms. How far do you travel from Platform B to Platform C? Each grid unit represents 5 m.



Think

Where's the right triangle?

The lengths of the legs of the right triangle are 15 and 30. There are two possibilities:



Let Platform B $(-30, -20)$ be (x_1, y_1) and Platform C $(-15, 10)$ be (x_2, y_2) .

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Use the Distance Formula.}$$

$$= \sqrt{(-15 - (-30))^2 + (10 - (-20))^2} \quad \text{Substitute.}$$

$$= \sqrt{15^2 + 30^2} = \sqrt{225 + 900} = \sqrt{1125} \quad \text{Simplify.}$$

$$\sqrt{1125} \approx 33.54101966 \quad \text{Use a calculator.}$$

You travel about 33.5 m from Platform B to Platform C.



Got It? 4. How far do you travel from Platform D to Platform E?



Lesson Check

Do you know HOW?

- \overline{RS} has endpoints $R(2, 4)$ and $S(-1, 7)$. What are the coordinates of its midpoint M ?
- The midpoint of \overline{BC} is $(5, -2)$. One endpoint is $B(3, 4)$. What are the coordinates of endpoint C ?
- What is the distance between points $K(-9, 8)$ and $L(-6, 0)$?

Do you UNDERSTAND?



MATHEMATICAL PRACTICES

- Reasoning** How does the Distance Formula ensure that the distance between two different points is positive?
- Error Analysis** Your friend calculates the distance between points $Q(1, 5)$ and $R(3, 8)$. What is his error?

$$\begin{aligned} d &= \sqrt{(1 - 8)^2 + (5 - 3)^2} \\ &= \sqrt{(-7)^2 + 2^2} \\ &= \sqrt{49 + 4} \\ &= \sqrt{53} \approx 7.3 \end{aligned}$$



Practice and Problem-Solving Exercises



A Practice

Find the coordinate of the midpoint of the segment with the given endpoints.

See Problem 1.

6. 2 and 4

7. -9 and 6

8. 2 and -5

9. -8 and -12

Find the coordinates of the midpoint of \overline{HX} .

10. $H(0, 0)$, $X(8, 4)$

11. $H(-1, 3)$, $X(7, -1)$

12. $H(13, 8)$, $X(-6, -6)$

13. $H(7, 10)$, $X(5, -8)$

14. $H(-6.3, 5.2)$, $X(1.8, -1)$

15. $H\left(5\frac{1}{2}, -4\frac{3}{4}\right)$, $X\left(2\frac{1}{4}, -1\frac{1}{4}\right)$

The coordinates of point T are given. The midpoint of \overline{ST} is $(5, -8)$. Find the coordinates of point S .

See Problem 2.

16. $T(0, 4)$

17. $T(5, -15)$

18. $T(10, 18)$

19. $T(-2, 8)$

20. $T(1, 12)$

21. $T(4.5, -2.5)$

Find the distance between each pair of points. If necessary, round to the nearest tenth.

See Problem 3.

22. $J(2, -1)$, $K(2, 5)$

23. $L(10, 14)$, $M(-8, 14)$

24. $N(-1, -11)$, $P(-1, -3)$

25. $A(0, 3)$, $B(0, 12)$

26. $C(12, 6)$, $D(-8, 18)$

27. $E(6, -2)$, $F(-2, 4)$

28. $Q(12, -12)$, $T(5, 12)$

29. $R(0, 5)$, $S(12, 3)$

30. $X(-3, -4)$, $Y(5, 5)$

Maps For Exercises 31–35, use the map below. Find the distance between the cities to the nearest tenth.

See Problem 4.

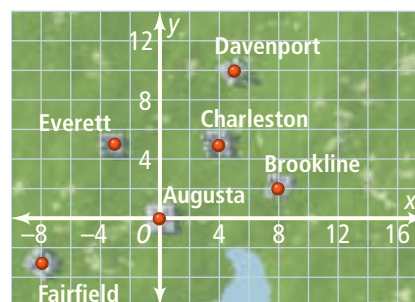
31. Augusta and Brookline

32. Brookline and Charleston

33. Brookline and Davenport

34. Everett and Fairfield

35. List the cities in the order of least to greatest distance from Augusta.



B Apply

Find (a) PQ to the nearest tenth and (b) the coordinates of the midpoint of \overline{PQ} .

36. $P(3, 2)$, $Q(6, 6)$

37. $P(0, -2)$, $Q(3, 3)$

38. $P(-4, -2)$, $Q(1, 3)$

39. $P(-5, 2)$, $Q(0, 4)$

40. $P(-3, -1)$, $Q(5, -7)$

41. $P(-5, -3)$, $Q(-3, -5)$

42. $P(-4, -5)$, $Q(-1, 1)$

43. $P(2, 3)$, $Q(4, -2)$

44. $P(4, 2)$, $Q(3, 0)$

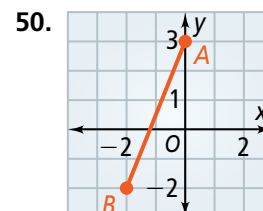
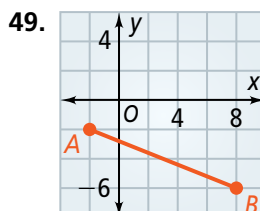
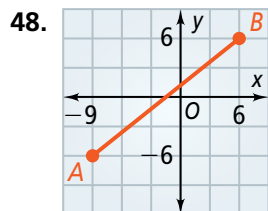
45. Think About a Plan An airplane at $T(80, 20)$ needs to fly to both $U(20, 60)$ and $V(110, 85)$. What is the shortest possible distance for the trip? Explain.

- What type of information do you need to find the shortest distance?
- How can you use a diagram to help you?

- © 46. **Reasoning** The endpoints of \overline{AB} are $A(-2, -3)$ and $B(3, 2)$. Point C lies on \overline{AB} and is $\frac{2}{5}$ of the way from A to B . What are the coordinates of Point C ? Explain how you found your answer.

47. Do you use the Midpoint Formula or the Distance Formula to find the following?
- Given points K and P , find the distance from K to the midpoint of \overline{KP} .
 - Given point K and the midpoint of \overline{KP} , find KP .

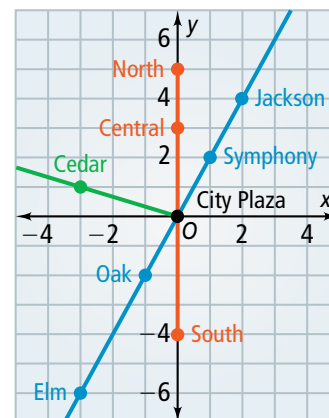
For each graph, find (a) AB to the nearest tenth and (b) the coordinates of the midpoint of \overline{AB} .



51. **Coordinate Geometry** Graph the points $A(2, 1)$, $B(6, -1)$, $C(8, 7)$, and $D(4, 9)$. Draw parallelogram $ABCD$, and diagonals \overline{AC} and \overline{BD} .
- Find the midpoints of \overline{AC} and \overline{BD} .
 - What appears to be true about the diagonals of a parallelogram?

Travel The units of the subway map at the right are in miles. Suppose the routes between stations are straight. Find the distance you would travel between each pair of stations to the nearest tenth of a mile.

- Oak Station and Jackson Station
- Central Station and South Station
- Elm Station and Symphony Station
- Cedar Station and City Plaza Station
- Maple Station is located 6 mi west and 2 mi north of City Plaza. What is the distance between Cedar Station and Maple Station?



- © 57. **Open-Ended** Point $H(2, 2)$ is the midpoint of many segments.
- Find the coordinates of the endpoints of four noncollinear segments that have point H as their midpoint.
 - You know that a segment with midpoint H has length 8. How many possible noncollinear segments match this description? Explain.



Challenge

58. Points $P(-4, 6)$, $Q(2, 4)$, and R are collinear. One of the points is the midpoint of the segment formed by the other two points.
- What are the possible coordinates of R ?
- © b. **Reasoning** $RQ = \sqrt{160}$. Does this information affect your answer to part (a)? Explain.

Geometry in 3 Dimensions You can use three coordinates (x, y, z) to locate points in three dimensions.

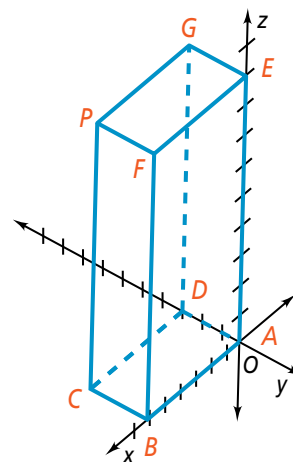
59. Point P has coordinates $(6, -3, 9)$ as shown at the right. Give the coordinates of points A, B, C, D, E, F , and G .

Distance in 3 Dimensions In a three-dimensional coordinate system, you can find the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) with this extension of the Distance Formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Find the distance between each pair of points to the nearest tenth.

60. $P(2, 3, 4), Q(-2, 4, 9)$ 61. $T(0, 12, 15), V(-8, 20, 12)$



Standardized Test Prep

SAT/ACT

62. A segment has endpoints $(14, -8)$ and $(4, 12)$. What are the coordinates of its midpoint?
 Ⓐ $(9, 10)$ Ⓑ $(-5, 10)$ Ⓒ $(5, -10)$ Ⓓ $(9, 2)$
63. Which of these is the first step in constructing a congruent segment?
 Ⓕ Draw a ray. Ⓗ Label two points.
 Ⓖ Find the midpoint. Ⓘ Measure the segment.

Short Response

64. The midpoint of \overline{RS} is $N(-4, 1)$. One endpoint is $S(0, -7)$.
 a. What are the coordinates of R ?
 b. What is the length of \overline{RS} to the nearest tenth of a unit?

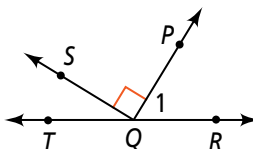
Mixed Review

Use a straightedge and a compass.

65. Draw \overline{AB} . Construct \overline{PQ} so that $PQ = 2AB$.
 66. Draw an acute $\angle RTS$. Construct the bisector of $\angle RTS$.

Use the diagram at the right.

67. Name $\angle 1$ two other ways.
 68. If $m\angle PQR = 60$, what is $m\angle RQS$?



Get Ready! To prepare for Lesson 1-8, do Exercises 69–72.

Complete each statement. Use the conversion table on page 837.

69. $130 \text{ in.} = \square \text{ ft}$ 70. $14 \text{ yd} = \square \text{ in.}$ 71. $27 \text{ ft} = \square \text{ yd}$ 72. $2 \text{ mi} = \square \text{ ft}$

◀ See Lesson 1-6.

◀ See Lesson 1-4.

◀ See p. 886.